Jornadas de Álgebra no conmutativa

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Abstracts

Primitivity of prime countable-dimensional regular algebras

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Let k be a field and let R be a countable dimensional prime von Neumann regular k-algebra. We show that R is primitive, answering a special case of a question of Kaplansky.

A family of solutions of the Yang-Baxter equation

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A new method to construct involutive non-degenerate set-theoretic solutions $(X^n,\, r^{(n)})$ of the Yang-Baxter equation from an initial solution (X, r) is given. Furthermore, the permutation group $\mathcal{G}(X^n, r^{(n)})$ associated to the solution $(X^n, r^{(n)})$ is isomorphic to a subgroup of $\mathcal{G}(X, r)$, and in many cases $\mathcal{G}(X^n, r^{(n)}) \cong \mathcal{G}(X, r)$.

Subconjuntos triviales maximales e ideales internos de anillos simples

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Let Q be a (not necessarily unital) associative ring. A nonempty subset S of Q is said to be a *trivial subset* if $S^2 = 0$. By Zorn's Lemma, any trivial subset is contained in a maximal one, which is obviously a zero-product subring. In this note we describe the maximal trivial subsets of a simple ring, by proving that the map $\mathcal{R} \mapsto \mathcal{R} \cap \operatorname{lann}(\mathcal{R})$ is a bijection from the set of all proper nonzero annihilator right ideals of Q onto the set of all maximal trivial subsets of Q. In the case that Q is Artinian, the maximal trivial subsets are of the form $eQ(1-e)$, where e is a nontrivial idempotente of Q, with $e_1Q(1-e_1) = e_2Q(1-e_2)$, for e_1, e_2 idempotents of Q, if and only if $e_1Q = e_2Q$. Finally, we show the relationship between maximal trivial subsets of a simple ring and the inner ideal structure of its associated Lie algebra.

On a generalization of Gorenstein projective modules

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We extend to the noncommutative case the concepts of G_C-projective module and dimension, and P_C-projective dimension (weakening the condition of C being semidualizing as well), prove that indeed they share the principal properties of the classical ones and relate this two new dimensions. We show that if C is what we call a faithfully generalized self-orthogonal module then both dimensions coincide on modules with finite P_C-projective dimension. In addition we give generalizations to some known results.

Biálgebras multiplicadoras débiles

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A non-unital generalization of weak bialgebra is proposed with a multiplier-valued comultiplication. Certain canonical subalgebras of the multiplier algebra (named the 'base algebras') are shown to carry coseparable co-Frobenius coalgebra structures. Appropriate modules over a weak multiplier bialgebra are shown to constitute a monoidal category via the (co)module tensor product over the base (co)algebra. As an application, the "free vector space"functor from the category cat of small categories to wmba, the category of (regular with right full comultiplication) weak multiplier bialgebras, is shown to possess a right adjoint, given by taking (certain) group-like elements. The talk is based on the work presented in [BoGTLC:wmbas].

References

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Algebraic properties of Clifford inner ideals

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Given a Lie ring L, an *inner ideal* B of L is a subgroup of L such that $[B[B,L]] \subseteq B$. If R is an associative ring with involution $*$, then $K := \mathrm{Skew(R,*)}$, the set of skew-symmetric elements of R in relation to $*$, is a Lie ring when equipped with the antisymmetric product $[x,y] := xy - yx$. We will say that a subgroup B of K is a *Lie inner ideal* of K if it is an inner ideal of K which in addition is abelian (i.e, such that $[B,B] = 0$).

In [B] the inner ideals of $[K,K]/(Z(R) \cap [K,K])$ are studied in the case when R is a simple Artinian ring (with some restrictions), and the conclusion is achieved (incompletely, a case was missing) that every Lie inner ideal is either of the form eKe^*, where e is an idempotent such that $e^*e = 0$, or is linearly generated by certain elements of fixed basis of R. We will focus our attention in this last kind of inner ideal, which we will call *Clifford*.

In [FGG] the study of inner ideals of $[K,K]/(Z(R) \cap [K,K])$ is generalized to the case when R is a simple ring with socle (with some restrictions), which has a geometric description, thanks to Kaplansky's Theorem, as the finite-rank operators of a vector space equipped with a nondegenerate bilinear form. This allows to describe Clifford inner ideals in geometric terms.

In [BrFG] we classify the Lie inner ideals of K when R is a centrally closed prime ring (with some restrictions). One of the appearing classes is again that of Clifford inner ideals, which we afford to describe in ring-theoretic terms by means of an idempotent e such that $ee^* = 0 = e^*e$. In this

talk we will prove the principal properties of Clifford inner ideals directly from its algebraic description, avoiding the use of Kaplansky's Theorem, so that we are not obliged to consider both spaces of vectors and operators.

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[FGG] A. Fernández López, E. García and M. Gómez Lozano. *Inner ideals of finitary simple Lie algebras*. J. Lie Theory **16**(1) (2006), 97–114.

Gradings and presheafs.

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We will show that gradings of a small category and marked Galois presheafs coincide. We infer that the universal cover exists and that the fundamental groupoid is the category of fractions.

Álgebras de caminos de Leavitt puramente infinitas simples de grafos de orden 4

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We classify the purely infinite simple Leavitt path algebras $L_K(E)$, where E is a directed graph with 4 vertices satisfying condition (SING). For this we will use firstly the characterization of isomorphy classes of purely infinite simple Leavitt path algebras in terms of the K_0-group, unit-order and a certain determinant related to the adjacency matrix of the graph (the so called algebraic KP-theorem). We have also found some elements providing information on the torsion part and on the torsion-free part of the K_0-group. We obtain a characterization of purely infinite simple Leavitt path algebras associated to finite graphs. It turns out that for a finite graph E, the algebra $L_K(E)$ is purely infinite simple if and only if it is simple and its socle vanishes. We have used computer-aided calculations for determining the orbits of the symmetric group on the set of binary matrices. This has reduced the amount of matrices that we have had to consider in our study. With this reduced set of matrices we have implemented the computation of a number of invariants which has finally provided us with the classification that we were looking for.

Cuasi-anillos de Mal'cev y descomposiciones LU de los automorfismos de un producto

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Given a category $\mathcal{C}$ with products, P. Pavešić [pav99] began the study of the group $\mathbf{Aut}_{\mathcal{C}}(X \times Y)$ by means of what can be considered upper and lower triangular automorphisms, that is, following the classical ideas of LU decomposition in Linear Algebra. This method has been proven successful to describe $\mathbf{Aut}_{\mathcal{C}}(X \times Y)$ in situations where very few morphisms between X and Y exist. Therefore, it is natural to ask whether or not these upper and lower triangular automorphisms generate the whole group of automorphisms in general [ark01, Problem 13th]. Here we study this question by considering $\mathcal{QR}ings$, the category of (graded) Mal'cev quasi-rings [malcev], which is a common framework for the study of (non necessarily commutative) rings and groups. We show that $\mathbf{Aut}_{\mathcal{C}}(X \times X)$ is generated by triangular automorphisms if and only if X is an abelian quasi-ring.

References

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Graded modules over classical simple Lie algebras

Simple graded modules over graded semisimple Lie algebras over an algebraically closed field of characteristic zero are studied. The invariants appearing in this classification are computed for the classical simple Lie algebras. In particular, some criteria are obtained to determine when a finite-dimensional irreducible module admits a compatible grading.

Un Criptosistema de clave pública sobre un anillo no conmutativo

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The aim of this work is to introduce a cryptographic application of an extension of the ring $\mathrm{End}(\mathbb{Z}_p \times \mathbb{Z}_{p^2})$. We introduce a cryptosystem that allows to exchange secrets through an insecure channel . We show how non-commutativity of the ring and the existence of a huge number of zero-divisors make it suitable for such cryptographic application.

The center of a Leavitt path algebra: the case of row-finite graphs.

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In this talk we introduce the basic results to understand how to achieve the center of a Leavitt Path algebra whose underlying graph is row finite. Some notions are essential, specifically, the extreme cycles. Vertices in these cycles (P_{ec}), jointly with vertices in cycles without exits (P_c) and line points (P_l) are proved key in order to determine the structure of the center of a Leavitt path algebra. Moreover, there is a fourth ingredient: the hereditary set described by those vertices whose tree has infinite bifurcations, denoted by P_{b^∞}. We show that nontrivial elements in the center of a Leavitt path algebra live in the ideal $I(P_{ec} \cup P_c \cup P_l)$ generated by these three sets of vertices. Leavitt path algebras whose graphs have a finite number of vertices are dense extensions of $I(P_{ec} \cup P_c \cup P_l)$. This means graphically that every vertex connects to $P_{ec} \cup P_c \cup P_l$. In the case of a prime Leavitt

path algebra, only one of these sets survives. It is also important for us to determine the extended centroid of a Leavitt path algebra over a finite graph.

As one of our main results, we will give bases for the homogeneous components of the center. In order to establish the structure of the center of a Leavitt path algebra over a row-finite graph we will need to add the new hereditary set P_{b^∞}. We finally present the structure theorem for the center using the above ingredients.

Algebraic applications in Coding Theory

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The aim of this lecture is to review some recent lines in the research in Coding Theory in which a key role is played by associative and non associative rings.

On Mono-correct Modules

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It is well known that for two sets E, F if there are injective maps $E \to F$ and $F \to E$ then there exists a bijection between E and F. This result called Cantor-Bernstein theorem is not true in module category. As an analogue, in module theory, an R-module M is said to be mono-correct if for any R-module N, if there are monomorphisms $M \to N$ and $N \to M$ then $M \cong N$. In our subject, we have studied interesting properties of mono-correct modules in the commutative case and we have given a new characterization of strongly π-regular rings.

Direct limits in the heart of a t-structure

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T-structures on triangulated categories were introduced in the early eighties by Beillison, Berstein and Deligne in their study of the perverse sheaves on an algebraic or an analytic variety (see [BBD]). The main discovery of this concept was the existence of an abelian category, called the heart of the t-structure, which allowed the development of a homology theory that is intrinsic to the triangulated category.

In [HRS], Happel, Reiten and Smal$\leq$, associated to each torsion pair in an abelian category $\mathcal{A}$, a t-structure in t he bounded derived category $D^b(\mathcal{A})$. This t-structure is the restriction of a t-structure in $D(\mathcal{A})$. We study the behavior of direct limits in the heart of this t-structure. For a given Grothendieck category $\mathcal{G}$ and a torsion pair $\mathbf{t} = (\mathcal{T}, \mathcal{F})$ in $\mathcal{G}$, we show that if the heart of the associated t-structure in the derived category $D(\mathcal{G})$ is AB5, then $\mathcal{F}$ is closed under taking direct limits. The reverse implication is true, even implying that the heart is a Grothendieck category, for a wide class of torsion pairs which include the hereditary ones, those for which $\mathcal{T}$ is a cogenerating class and those for which $\mathcal{F}$ is a generating class. The results allow to extend well-known results by Buan-Krause, Bazzoni and Colpi-Gregorio to the general context of Grothendieck categories and to improve some results of (co)tilting theory of modules.

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Connections Of Roots In Split Structures

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This communication is a summary of the second author's Doctoral Thesis under the supervision of the first author. Techniques of connections of roots for different classes of split algebras and superalgebras are studied. The objective is to study the structure of these objects.

On rings characterized by properties of sum classes of their categories of modules.

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Let R be a ring. A left R-module M is said to be co-hopfian (resp. hopfian) if every injective (resp. surjective) endomorphism of M is an automorphism of M. M is said to be weakly co-hopfian (resp. generalized hopfian) if every injective (resp. surjective) endomorphism of M is essential (resp. superfluous). Finally, M is said to be Dedekind finite if M is not isomorphic to any proper direct sumand of itself. In this talk we prove that for a duo ring R the following properties are equivalent:

1.- R is an artinian principal ideal ring.

2.- Every co-hopfian R-module is artinian.

3.- Every hopfian R-module is noetherian.

4.- Every weakly co-hopfian R-module is finitely generated.

5.- Every generalized hopfian R-module is finitely generated.
6.- Every R-module is a direct sum of cyclic submodules.

The universal enveloping algebra of the Witt algebra is not noetherian

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We outline our recent proof (with Walton) that the universal enveloping algebra of the Witt algebra is not noetherian, settling a long-standing question. The proof uses geometric techniques from our classification of birationally commutative projective surfaces

Quasigroups de Hopf y productos cruzados

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In this work we introduce the notion of cleft right H-comodule algebra (A, ϱ_H) over a Hopf quasigroup H and expose necessary and sufficient conditions such that $A \simeq A_H \sharp_{\sigma_{A_H}} H$ as algebras, being A_H the subalgebra of coinvariants of A and σ_{A_H} a suitable cocycle. It extends to the non associative setting the result given by Blattner, Cohen and Montgomery for projections of Hopf algebras with coalgebra splitting.